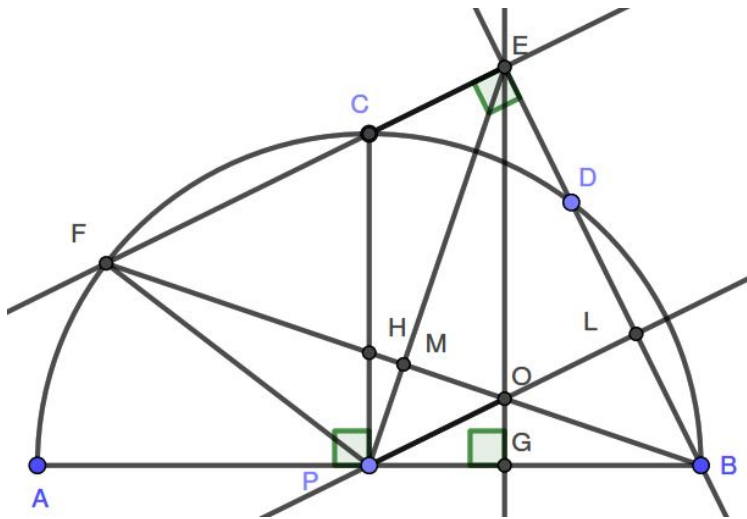## Solution:

AB is the diameter of semicircle.

Let P be the centre of semicircle.

Construction: JOIN CP, Join FP, Join EP

FP intersect CP at H and EP at M

Given C is the mid point of Arc AB.

Hence $CP \perp AB$ ----------------(1)

Here CP is the perpendicular bisector of AB.

Given that $EG \perp AB$ --------------(2)

From (1) and (2) $CP \parallel EG$ -------------(3)

$\angle BFC = \dfrac{1}{2} \times \angle CPB = \dfrac{90^0}{2} = 45^0$ (Segment angle is half of central angle.)

Hence $\triangle BEF$ is a right isosceles triangle.

FE = EB ----------------------(4)

In quadrilateral EFPB, PF = PB (radii) and EF = FB

So, EFPB is a kite,

$\Rightarrow EP \perp FB$ (diagonals are perpendicular in KITE)

Now in triangle EPB, $EG \perp PB$ and $BM \perp EP$ $(EP \perp FB)$

So, O is the orthocentre of triangle EPB.

Hence Extended PO intersect EB at L

Hence $PL \perp EB$ -------------(5) (As O is orthocentre of triangle EPB)

Given $FE \perp EB$ -------(6)

From (5) and (6) $FE \parallel PL$ $\Rightarrow PO \parallel CE$ ------- (7)

From (3) and (7) CPOE is a parallelogram.

Hence EO = CP (radius)

$\Rightarrow EO = \dfrac{1}{2} \times AB$ (Proved) (As AB is the diameter)

DR SHYAM SUNDAR AGRAWAL

RANI RASHMONI HIGH SCHOOL (H.S.)

104/A S N BANERJEE ROAD

TALTALA – KOLKATA

WEST BENGAL – 700014